

# Evaluating baseball bat performance

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## Abstract

Selected methodologies currently used to assess baseball bat performance were evaluated through a series of finite element simulations. Results of the comparison show that current test methods contradict one another and do not describe the performance advantage of modern hollow bats over solid wood bats. The discrepancy was related to the way performance was quantified and the way the bat was tested. Performance metrics that do not consider a bat's mass moment of inertia (MOI) were observed to underestimate the hitting performance of light weight bats. A bat's centre of percussion was observed to be an unreliable indicator of its sweet spot (i.e. impact location providing the maximum hit ball speed). Bat performance was found to be sensitive to the relative impact speed between the bat and ball. From these observations three recommendations concerning bat performance were made: (1) performance should be measured at relative speeds between the bat and ball that are representative of play conditions; (2) the ball should impact the bat at its experimentally determined sweet spot; and (3) performance should be quantified from ball and bat speeds before and after impact. Using current test methods, an aluminium bat had a 0.9% higher performance over wood (maximum), while using the proposed recommendations the difference was 3.8% (average). The variation in the relative performance over three test conditions reduced from  $\pm 4\%$  using current test methods to  $\pm 0.3\%$  when the above recommendations were followed.

*Keywords:* baseball bat, hitting performance, bat test methods

## Introduction

The acceptance of nonwood bats in amateur baseball has presented some challenges to the game. The interest in alternative materials initially concerned the expense associated with frequent wood bat failure, Crisco (1997). It was soon discovered, however, that hollow bats hit the ball differently than solid wood bats. This observation motivated many bat producers to optimize their products for performance rather than durability.

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The increased performance has had many side-effects on the game, including safety, cost, and competitive balance.

There is a strong desire among amateur leagues to limit the performance of modern bats. While this idea may appear unique to baseball, it is achieved to some degree in many sports through control over ball performance, USGA (1998), ITF (2001). Since restrictions have not placed limits on the composition the bat, the focus has been to ensure a uniform performance among bats, independent of their material.

Assessing bat performance has proved to be a non-trivial task. The motion of the bat involves complex three dimensional translation and rotation. Given the complexities of the bat motion and interaction

with the ball, it has been unclear how performance might be assessed in a controlled laboratory environment that would represent response in play.

Many amateur leagues place controls on bat weight since it is easily measured, understood and most hollow bats are lighter than their solid wood counterparts. It has been observed, however, that the rotational inertia of the bat is important to its response, Crisco (1997). Test methods have been developed to simulate bat motion and represent progress toward assessing bat performance. The current study will show, however, that current test standards may not correctly describe the hitting performance that may be observed in play. The comparison is made using a computational model, that has been verified experimentally, Shenoy *et al.* (2001). This approach allows control over bat and ball position, as well as the properties whose variation can hinder experimental investigations.

## Background

An experimental comparison of the test methods currently used to assess bat performance is outside the scope of this study. Recent advances in computational modelling of the ball-bat impact indicate that such a comparison may not be necessary. It has been observed, for instance, that the hitting performance of a bat is largely independent of its constraint, Nathan (2000), Smith *et al.* (2000). This may be qualitatively explained by considering the relatively short contact duration between the bat and ball ( $\sim 1$  ms) and the load that may be transmitted to the bat from its constraints during this time. Constraint conditions represent a primary difference between testing machines. If performance may be considered independent of constraint, test results between machines may be compared directly. Bat constraint does have a large effect on the loads transmitted to the bat after the impact is complete, however, and should be carefully considered in bat durability studies, Axtell *et al.* (2000).

A comparison of test methods will typically consider each test system and its interaction with a specimen. Given the small effect of constraint for the case of bat performance, the focus of this

comparison will be directed toward the motion of the bat and ball, rather than entire systems (i.e. the device used to create the bat motion).

An explicit dynamic finite element model (FEM) has been constructed to simulate the impact between a bat and ball (using LS Dyna, version 950) as shown in Fig. 1. Details of the model and its verification with experiment may be found elsewhere, Smith *et al.* (2000), Shenoy *et al.* (2001). A unique aspect of the model involves an approach used to accommodate energy losses associated with elastic colliding bodies. This was accomplished by modelling the ball as a viscoelastic material with high time dependence. The viscoelastic response of the ball was characterized through quasi-static tests and high-speed rigid-wall impacts. This approach found excellent agreement with experiment for comparisons involving rebound speed, contact force and contact duration. The model also captured the ball's speed-dependent coefficient of restitution, which decreased with increasing impact speed.

The model was verified by simulating a dynamic bat-testing machine involving a swinging bat and a pitched ball, Shenoy *et al.* (2001). The good agreement between the model and experiment for numerous bat and ball types, impact locations and speeds, as well as bat strain response indicate the model has broad application in accurately predicting bat performance.

With a model in place, comparisons between currently used bat-performance test methods may be conducted. Virtual testing of this type has the advantage of complete control over the bat and ball properties. These properties are difficult to

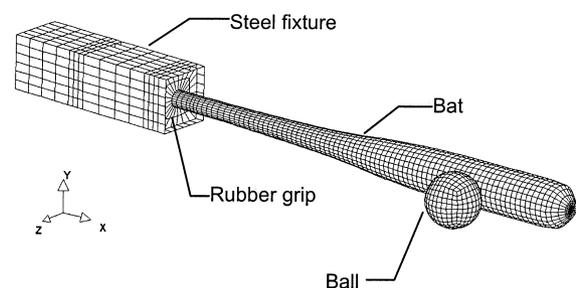


Figure 1 Diagram of finite element ball-bat impact model.

monitor experimentally, and can affect experimental results given the subtle, but significant, differences among balls and bats. Thus, performance measures (experimental or theoretical) are most useful when provided on a relative basis with constant test conditions and using an accepted benchmark.

### Bat properties

In the following comparisons, commercially available solid-wood and hollow-aluminium bats are considered. Each bat had a length of 860 mm (34 in). Their mass properties were measured and may be found in Table 1. The wood bat is slightly heavier and consequently exhibits a larger MOI. While this is typical, it should not be considered a rule. The hollow structure of metal bats allows manipulation of their inertia in nonobvious ways. The profiles of the two bats were similar, but not identical. The effect of bat profile for the normal and planar impacts considered here was not significant. The properties used for the ball were found from dynamic tests of a typical collegiate certified baseball. Details of the material properties used for the ball and bats may be found elsewhere, Shenoy *et al.* (2001).

### Bat motion

The motion of a swinging bat, as observed in play, may be described by an axis of rotation (not fixed relative to the bat), its rotational speed and location in space. The axis of rotation and its orientation move in space as the bat is swung and its rotational velocity increases. Thus, three-dimensional translation and rotation are required to describe the motion of a bat swung in play.

Determining a bat's hitting performance requires only a description of its motion during the instant

**Table 1** Mass properties of a solid wood and hollow metal bat

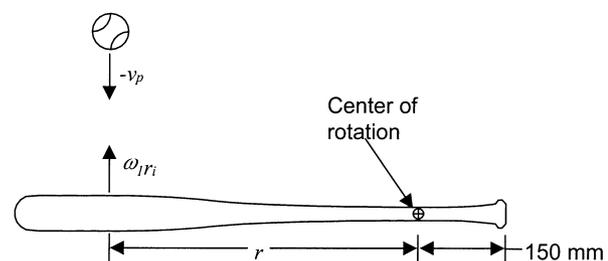
Bat	Mass (g)	C.G. (mm)	MOI (kg m <sup>2</sup> )
Ash	906	429	0.209
Aluminium	863	418	0.198

MOI and centre of gravity (C.G.) are measured from the bat's centre of rotation.

of contact with the ball. The motion over this short time period has been observed as nearly pure rotation, with the fixed centre of rotation located near the hands gripping the bat, Eggeman & Noble (1982). The exact motion of the bat will obviously vary from player to player. For the study at hand, a fixed centre of rotation, located 150 mm from the knob end of the bat was used, and is shown in Fig. 2. The impact location with the ball,  $r_i$ , is measured from the centre of rotation. The bat's rotational speed before impact is designated  $\omega_1$ , while the ball's pitch speed is  $v_p$ . (Pitch speed is taken here as a negative quantity to maintain a consistent coordinate system.)

### Test methods

To test a bat one must assume a representative motion (typically rotation about a fixed centre), a bat and ball speed, a performance measure, and an impact location. From observations of amateur and professional players, typical bat swing speeds have been observed to range from 34 to 48 rad s<sup>-1</sup>, Crisco *et al.* (2000). Some practitioners of the game believe this number should be higher, but experimental measurements have not shown this, Fleisig *et al.* (1997) and Koenig *et al.* (1997). Pitch speed is more easily measured (often occurring live during a game) and may range from 20 m s<sup>-1</sup> to 40 m s<sup>-1</sup>. Thus, in a typical game, the relative speed between the point of contact on the bat and ball may vary by a factor of three. Of primary interest in bat performance studies is the maximum hit ball speed. To this end, tests are usually conducted toward the higher end of these relative speed ranges.



**Figure 2** Schematic of assumed bat motion during impact with a baseball.

Three test methods are commonly used to evaluate bat performance. The first method involves pitching a ball toward a swinging bat, NCAA (1999). This is the most difficult test to perform of the three methods and requires accurate positioning of the bat and ball, timing of their release and control of their speed. The NCAA currently uses this type of test to certify bats for collegiate play.

A second method involves pitching a ball toward an initially stationary bat. This has been accepted as an ASTM standard, ASTM (2000). This test is much simpler to perform than the NCAA test, since bat speed and timing do not need to be controlled. The method requires measurement of the bat speed after impact, however, which can be difficult to discern from its vibrating response.

In the current FEM bat vibration also hindered the determination of its after impact speed. The problem was avoided by consideration of a momentum balance of the ball-bat system as

$$I\omega_1 + mv_p r_i = I\omega_2 + mv_b r_i, \quad (1)$$

where  $I$  is the mass moment of inertia of the bat about its centre of rotation,  $\omega_2$  is the bat rotational velocity after impact, and  $m$  and  $v_b$  are the mass and hit speed of the ball, respectively.

A third method for testing bats involves swinging it toward an initially stationary ball. This method has not yet been incorporated into a test standard, but it is often used to study bat performance unofficially. It shares similar advantages of simplicity with the ASTM method over the NCAA method. It does require fabricating a device to swing a bat, which may be more costly than the bat support fixture and pitching machine used with the ASTM method.

### Current performance metrics

A bat's performance should be determined in a way that allows comparison with other bats and test methods. Three metrics are commonly used to quantify bat performance. The first simply uses the measured hit-ball speed obtained from a test

directly. While this value is of primary interest in play, its use as a performance metric in the laboratory has several limitations. First, it is sensitive to variations in the pitched ball and bat swing speeds, so that this variation will cause scatter in the performance metric. The second limitation concerns the momentum of the swinging bat. If all bats are tested at the same pitch and swing speed, then the bats with greater inertia will generally produce higher hit-ball speeds. The opposite trend is typically observed in play, however, where the lighter, low inertia, hollow bats typically hit the ball further.

The NCAA method uses what is termed a Ball Exit Speed Ratio (or *BESR*) to quantify bat performance at its experimentally determined sweet spot,  $r = r_s$ . It is a ratio of the ball and bat speeds and is defined as

$$BESR = \frac{v_b - 1/2(\omega_1 r_i + v_p)}{\omega_1 r - v_p}, \quad (2)$$

where  $r_i$  is the impact location on the bat, and  $v_b$  is the hit ball speed. It is used to normalize the hit-ball speed with small variations that inevitably occur in controlling the nominal pitch and swing speeds. It may be found from the coefficient of restitution,  $e$ , as  $BESR = e + 1/2$  (with  $\omega_1 = \omega_2$ ) where for the ball-bat system

$$e = \frac{\omega_2 r_i - v_b}{v_p - \omega_1 r_i}. \quad (3)$$

The assumption of constant swing speed can lead to erroneous results if bats with different MOI's are being compared. A lighter bat will have a slower swing speed after impact with a ball than a heavy bat. Since  $\omega_2$  would appear in the numerator of Eq. (2) as a negative contribution, the *BESR* produces a lower measure of bat performance for light bats than would occur if  $\omega_1 \neq \omega_2$ . The *BESR* is nevertheless popular because it avoids the experimentally difficult task of determining  $\omega_2$ .

The performance metric used by the ASTM method is termed the Bat Performance Factor (or *BPF*) and is found at the bat's centre of percussion,  $r = q$ , defined as

$$q = \frac{k_o^2}{\bar{r}} \quad (4)$$

where  $k_o^2$  is the bat's radius of gyration and  $\bar{r}$  is the location of its centre of gravity, both in relation to the centre of rotation, Meriam & Kraige (1997). The *BPF* considers initial variation in speed, the momentum of the bat and ball, as well as variations that may occur between balls. It is defined as the ratio of the ball-bat coefficient of restitution,  $e$ , and the coefficient of restitution of the ball used for testing,  $e_b$ , as

$$BPF = \frac{e}{e_b} \quad (5)$$

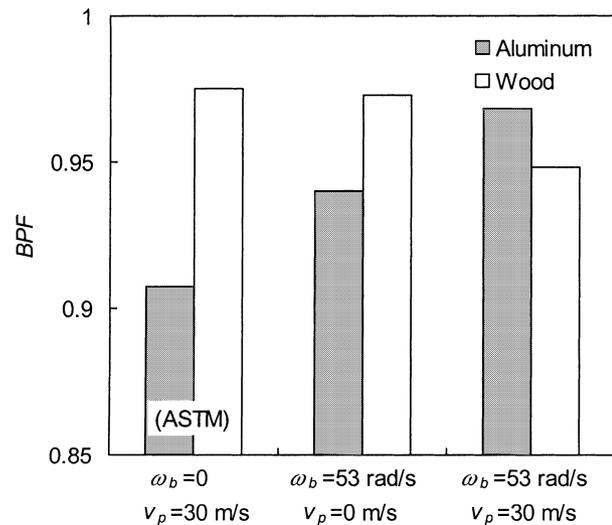
While this metric accounts for the primary factors affecting bat performance, it should not be considered as a quantity independent of test conditions. The response of the bat and ball are known to be rate dependent, for instance. It would be inappropriate therefore to compare the performance of two bats under different impact speeds with any of these performance metrics.

The quantitative values of the *BESR* and *BPF* for many bats are similar. This coincidence should not imply that they should be compared directly, however. In the following, bat performance will be discussed on a relative basis as

$$P = \frac{P_a - P_w}{P_w} \quad (6)$$

where  $P$  is the performance measure (*BESR* or *BPF*) and the subscript  $a$  and  $w$  represent a wood or aluminium bat, respectively.

Wood and metal bats are compared in Fig. 3 using the *BPF* for three circumstances involving the following initial conditions: stationary bat (ASTM), stationary ball, and moving ball and bat. The *BPF* was found at each bat's centre of percussion,  $q$ , as suggested by the ASTM standard. The speeds used here are near the range observed for play, and are the same that the NCAA method uses for certification. The pitch speed is  $3 \text{ m s}^{-1}$  higher than the ASTM recommended speed, although the ASTM method does have provision for 'elevated speeds'. The relative performance of the wood and metal



**Figure 3** The predicted bat performance factor found at the bats' respective centre of percussions for three test initial conditions.

bats is observed to vary greatly between the three test conditions. The metal bat is expected to have a higher performance, but this is only observed for the moving bat and ball case. This does not imply that the wood bat has a higher hitting performance, but illustrates the effect that test conditions can have on bat performance.

The *BESR* of each bat was found at its sweet spot (as recommended by the NCAA) and is shown in Fig. 4. A large difference in the relative performance of the wood and metal bats is again apparent, and is extreme for the initially stationary bat. This is a result of using a constant bat speed in the *BESR*, which is a poor approximation for the initially stationary bat case. Considering the ASTM and NCAA tests from Figs 3 and 4, we have  $P = -7\%$  and  $P = 0.9\%$ , respectively. This study will show that both of these test methods underestimate the relative hitting performance of the metal bat.

Another example illustrating the differences between the *BESR* and *BPF* is given in Fig. 5 for a hypothetical metal bat as a function of wall thickness (percentage of nominal). Of significance in the figure is the opposite effect that wall thickness (or *MOI*) has on the two metrics for

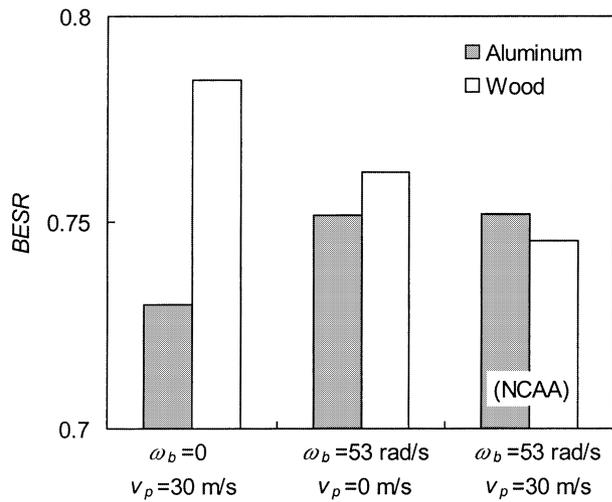


Figure 4. The predicted ball exit speed ratio found at the bats' respective sweet spots for three test initial conditions.

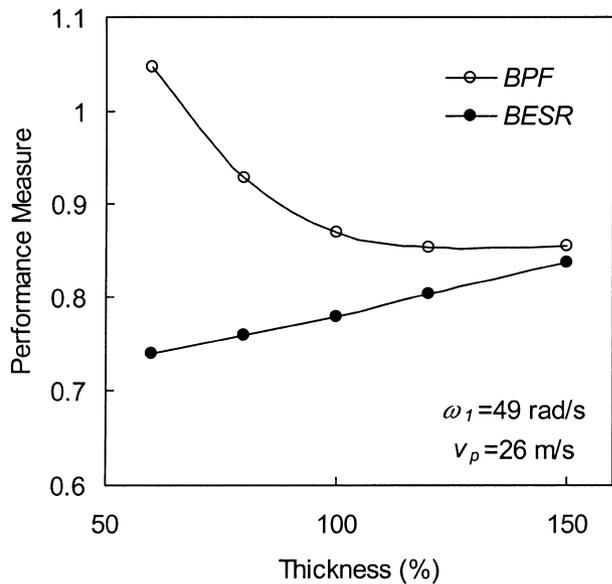


Figure 5. Bat performance measures as a function of wall thickness of an aluminium bat.

thin walled bats. The *BESR* and *BPF* begin to converge for heavy bats. This may be expected since the assumption of  $\omega_1 = \omega_2$  used in the *BESR* improves as the bat MOI increases. The change in wall thickness considered here is unrealistically large, primarily for demonstration purposes. The large *BPF* with decreasing wall thickness shown in

Fig. 5, for instance, would be limited in practice by barrel yielding, which has not been included in the model.

### Impact location

As observed by any player of the game of baseball, the hit-ball speed is dependent on its impact location with the bat. Many believe that a bat's sweet spot coincides with its centre of percussion,  $r = q$ , defined as the impact location that minimizes the reaction forces at its fixed centre of rotation. As will be shown below, they are offset slightly.

The difference between the centre of percussion and the sweet spot may be partially explained by considering the momentum balance of the bat-ball impact. Combining Eqs (1) and (3) to eliminate  $\omega_2$  produces an expression that may be solved for  $v_b$ . The sweet spot,  $r = r_s$ , can be obtained by minimizing this result with respect to  $r_s$ , equating to zero, then solving for  $r_s$ , as

$$r_s = \frac{v_p}{\omega_1} + \sqrt{\frac{I}{m} + \left(\frac{v_p}{\omega_1}\right)^2} \quad (7)$$

In the above expression,  $v_p < 0$ , so that  $r_s = 0$  when  $\omega_1 = 0$ . Thus, under rigid-body-motion assumptions, an initially stationary bat will impart the highest hit-ball speed when impacted at its fixed constraint. This may be expected since an initially stationary bat has no rotational energy to impart to the ball, and any other impact location would transfer energy from the ball to the bat. Comparison of Eqs (4) and (7) demonstrates that  $r_s \neq q$ . While rigid body motion is clearly an oversimplification of the bat-ball impact, Eq. (7) suggests that the sweet spot of a bat may not be fixed but depend on the initial bat and ball speed. The computational model described above was used to investigate the effect of a nonrigid bat's rotational speed on its sweet spot by simulating impacts along its length. This was done at 50-mm intervals over the range of 350 mm  $> r >$  650 mm. This produced a curve of hit-ball speed vs. impact location, as shown in Fig. 6. The location of the maximum of this curve yielded  $r_s$ .

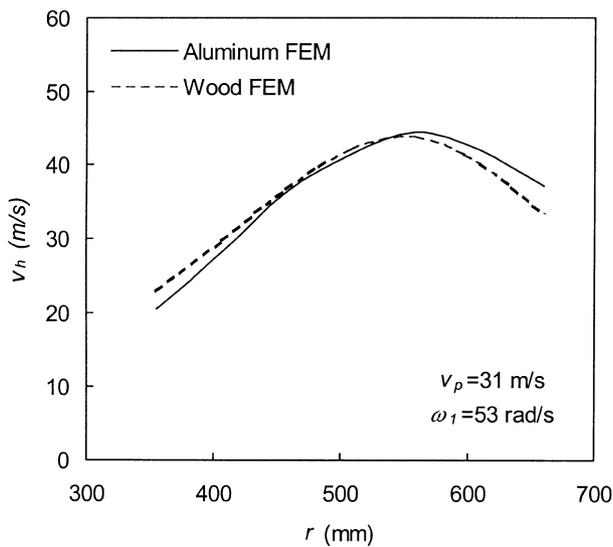


Figure 6 Hit ball speed as a function of impact location for a wood and aluminium bat.

A comparison of the sweet spots, as a function of swing speed, was obtained from the momentum balance, Eq. (7) and the FEM for the two bat types described in Table 1, and may be found in Fig. 7. The momentum balance clearly overestimates the magnitude of the dependence of  $r_s$  on  $\omega_1$ . A dependence is observed in the finite element results, where the sweet spot is observed to move up to 50 mm between the stationary and swinging conditions. A similar dependence was observed with pitch speed (but not shown here for brevity), where the sweet spot was observed to move up to 35 mm between stationary and pitched ball conditions. The sweet spot locations from the FEM are contrasted with the centre of percussion found from Eq. (4) for the metal and wood bats, in Fig. 8. A notable observation from Fig. 8 concerns the relative locations of  $r_s$  and  $q$  for the wood and metal bats. The centre of percussion for the metal bat is 25 mm inside its sweet spot, while the centre of percussion for the wood bat lies only 2 mm inside of its sweet spot. (The centre of percussion of the metal bat is observed to be 10 mm inside that of wood, a trend that was consistent from a group of 12 bats including bats of solid wood, and hollow metal and composite construction.) Thus, tests

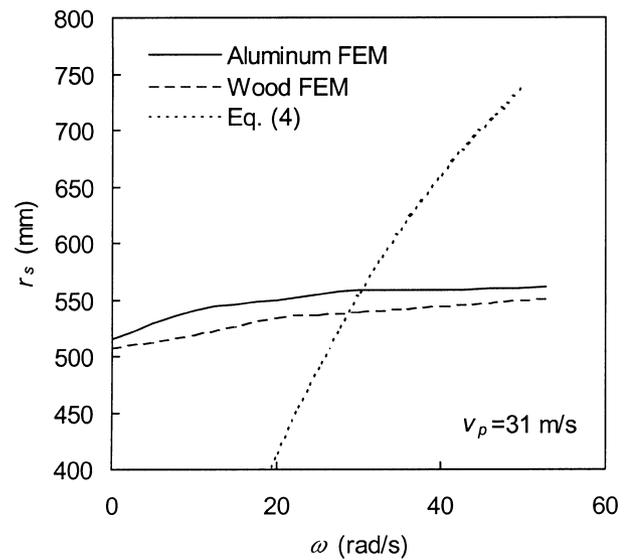


Figure 7 Effect of bat rotational speed on the location of its sweet spot.

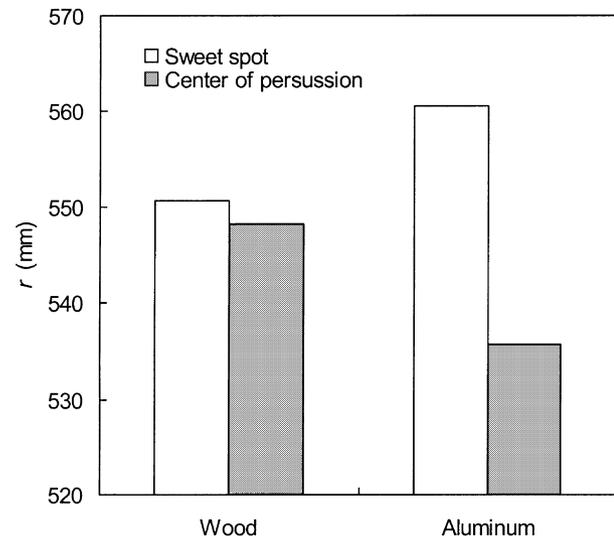


Figure 8 Comparison of the location of the sweet spot and centre of percussion for a wood and aluminium bat. (Sweet spot found using  $v_p = 31 \text{ m s}^{-1}$  and  $\omega = 53 \text{ rad s}^{-1}$ .)

which consider impacts of bats at  $r = q$  may provide an unfair comparison, and do not represent their relative hit-ball speed potential. The utility of using rigid body dynamics to determine an appropriate impact location for dynamic bat testing appears dubious. Similar observations could be

made for tests that consider a fixed impact location, independent of bat composition.

### Improved performance measure

While the contradictions between current test methodologies are unfortunate, the following will show that they may be improved and simplified to provide a more accurate description of bat performance. In selecting a numerical quantity to describe bat performance, the results of Fig. 5 show that the effect of bat MOI can be important and is described by the *BPF*.

The *BPF* was modified by computing it at each bat's sweet spot as shown in Fig. 9. The variations between the test conditions are observed to decrease, where for two of the three conditions  $P = 3\%$ . For the third condition, however,  $P = -6\%$ . The dependence of bat performance on the test condition is obviously undesirable and is addressed below.

As observed in Fig. 5, a hollow bat's hitting performance increases with decreasing wall thickness. It is possible that this phenomenon is related to an increased radial barrel deformation that would occur with a thinner wall. This may be explained in terms of ball deformation and impulse.

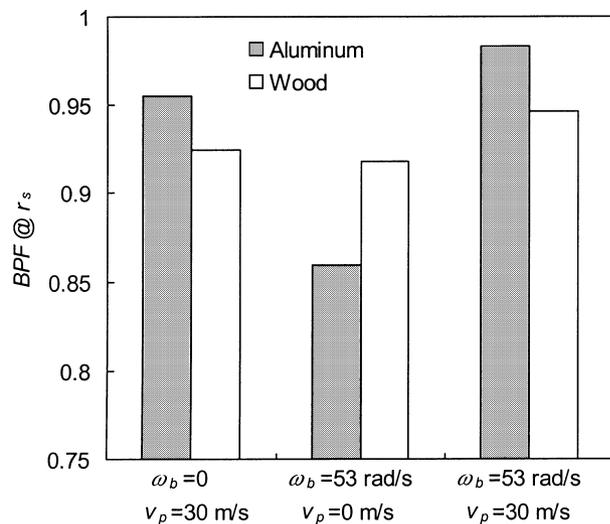


Figure 9 The predicted bat performance factor found at the bats' respective sweet spots for three test initial conditions.

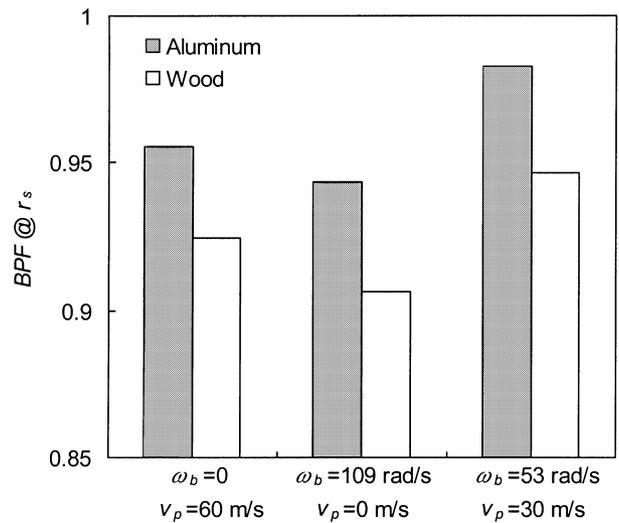


Figure 10 The predicted *BPF* found at the bats' respective sweet spots using a constant relative impact speed between the ball and the bat for three test initial conditions.

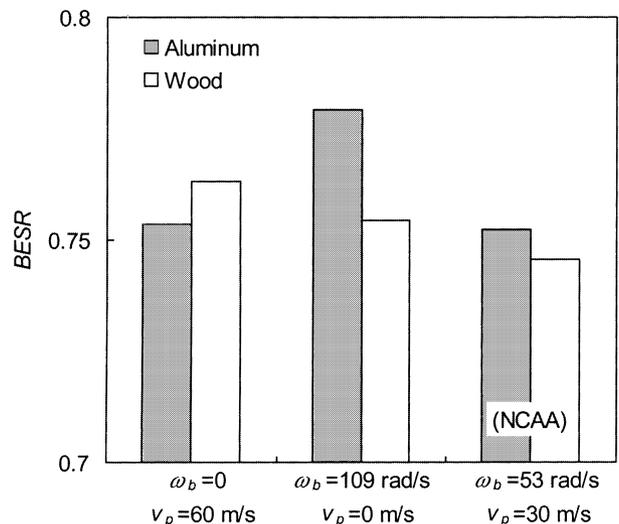


Figure 11 The predicted *BESR* found at the bats' respective sweet spots using a constant relative impact speed between the ball and the bat for three test initial conditions.

Impact studies of various types of baseballs have shown its coefficient of restitution to decrease with increasing impact velocity (or deformation). Impacts involving a deforming hollow barrel would reduce the amount of ball deformation, and potentially lengthen the duration of contact between the

ball and bat. Thus, hollow bats have the potential of reduced energy loss during ball impact and imparting greater impulse to the ball.

From the foregoing, and in the context of bat evaluation, it is desirable to compare bats under impact conditions that are similar to play conditions. In the comparisons of Figs 3, 4, and 9, the relative velocity of the bat and ball varied between  $30 \text{ m s}^{-1}$  and  $60 \text{ m s}^{-1}$ . A constant relative speed of  $60 \text{ m s}^{-1}$  (comparable to play conditions) may be accomplished by increasing the pitch and swing speeds of the initially stationary bat and ball tests, respectively. This provides a similar impact force on the bat for all three conditions, and reduces performance variation due to barrel deformation. The results of impacts at the bat's sweet spot are shown in Figs 10 and 11, for the *BPF* and *BESR*, respectively. The magnitude of the *BPF* and *BESR* continue to depend on the test condition. The variation in the relative difference between the wood and metal bats for the three conditions is reduced considerably, however ( $P = 3.8 \pm 0.3\%$  and  $P = 1.0 \pm 2.3\%$  for the *BPF* and *BESR*, respectively). As was noted earlier, the performance difference between the metal and wood bats is lower and the variation is higher using the *BESR* than that found with the *BPF*.

## Conclusions

A computational model has been presented and used to evaluate test methods from industry, the NCAA and the ASTM in assessing bat performance. A comparison between a representative wood and aluminium bat shows that current standard test methodologies may not accurately represent a bat's hitting performance in play. Recommendations for improving bat assessment include accounting for the bat's inertia to quantify bat performance, evaluating a bat at its experimentally determined sweet spot (as done by the NCAA) and using relative impact speeds between the bat and ball that are representative of play conditions. By following these recommendations it was shown that the relative performance of a metal bat over wood was nearly four times that found using current test

methods. The variation in performance over three test conditions using the above recommendations was 13% of that occurring from current test methods. These results also suggest that bat performance can be accurately determined using a simplified test involving either an initially stationary bat or ball.

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