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Progress and challenges in numerically modelling solid sports balls with application to softballs

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Progress and challenges in numerically modelling solid sports balls with application to softballs

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Abstract

Much of the work surrounding finite element simulation of bat–ball impacts has focused on techniques describing the ball. Determining the accuracy of these models has been hindered by challenges in experimentally characterizing the ball's response. In the following, dynamic mechanical analysis and an instrumented impact test were used to characterize the solid ball at deformation rates representative of play. The ball was described in the numerical model as a linear viscoelastic material. It was observed that a Prony series model based on small deformation dynamic mechanical analysis did not provide sufficient energy loss upon impact, while a simpler Power Law model, fitted to large deformation data, described the measured energy loss and impact force over a range of speeds. Results of a parametric study are presented as a guide towards tailoring the parameters of the Power Law model to match the measured energy loss and impact force. Discrepancies observed between the experiment and the numerical model suggest that the ball response should be characterized in environments closely resembling game conditions.

Keywords: *Softball, viscoelasticity, finite element analysis, ball characterization*

Introduction

The design of solid ball-impact sports equipment, such as for baseball, softball, golf, and cricket, is largely empirical, in spite of marked advances that have been made in numerical impact models. The club or bat is nearly elastic (Nathan, 2000) and readily described using finite elements. The ball, however, presents formidable challenges including non-linear response, rate dependence, and energy dissipation.

Several studies have modelled balls in an effort to examine safety. Crisco (1997) used quasi-static ball-compression tests to characterize baseball properties. The subsequent model was used to predict impact responses of the head and chest. The ball model assumed a perfectly elastic collision.

Bathke (1998) used ABAQUS to build a multi-layered model of a baseball impacting a rigid steel wall at several speeds. Although each material was modelled as linearly elastic, the coefficient of restitution (*COR*) was observed to decrease linearly with increasing pitch speed. Bathke attributed the loss in energy to internal vibrations of the baseball. Bathke's *COR* values were high, but the dependence of the coefficient of restitution on speed agreed with the experimental data of Hendee and colleagues (Hendee, Greenwald, & Crisco, 1998).

Mustone and Sherwood (1998) modelled a baseball using LS-DYNA. The Mooney-Rivlin material model (Ogden, 1984) was characterized from a quasi-static load-displacement curve. The finite element model was calibrated to match the *COR* at 27 $m \cdot s^{-1}$ (60 mph) for a baseball. The subsequent ball model was used to describe bat performance at a relative speed of 63 $m \cdot s^{-1}$ (140 mph).

Shenoy (2000), Sandmeyer (1994), Axtell (2001), and Nicholls (2005) modelled a baseball using a viscoelastic material in LS-DYNA, defined from a time-dependent shear modulus as

$$G(t) = G_\infty + (G_0 - G_\infty)e^{-\beta t}, \qquad (1)$$

where G_∞ and G_0 are the long-term and instantaneous shear moduli, respectively, t is time, and β is the decay constant. Equation (1) is often referred to as a Power Law viscoelasticity model.

Sandmeyer (1994) used Equation (1) to model softballs and baseballs impacting a rigid wall. The viscoelastic parameters were adjusted until his

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model agreed with the measured *COR* and contact times.

Shenoy (2000) used eight-noded solid elements to model a baseball. The ball was characterized quasi-statically and by measuring the impact force from a flat, rigid-wall *COR* test. The constant G_∞ was found using

$$G_\infty = \frac{E}{2(1+v)}, \quad (2)$$

where the elastic modulus, E, and Poisson's ratio, v, were obtained from static tests. The constants G_0 and β were adjusted until the *COR* and impact force of the model and experiment agreed.

Nicholls (2005) argued that the decay constant, β should be as close to the contact time as possible. The long-term shear modulus, G_∞, was determined from quasi-static compression tests and G_0 was found through iteration. The finite element results showed that the baseball *COR* increased with increasing pitch speed. This finding was contradictory to the experimental work of Adair (2002), Axtell (2001), Chauvin and Carlson (1997), Cross (2000), Hendee et al. (1998), and Drane & Sherwood (2004). Conversely, peak force increased and contact time decreased with increasing pitch speed, which was in line with experimental results.

Mase and Kersten (2004) modelled the *COR* and contact time of a golf ball using dynamic mechanical analysis (DMA). Specimens were machined from the golf-ball cover and rubber core. Relaxation tests were conducted as a function of temperature, to which a master curve was generated and fitted to a Prony series of the form

$$g(t) = \sum_{i=1}^{6} G_i e^{-\beta_i t}, \quad (3)$$

where $g(t)$ is the relaxation function and G_i and β_i are modulus and time constants, respectively. The Prony series was used in LS-DYNA to model the behaviour of a golf ball impacting a rigid steel wall, which agreed with experimental *COR* and contact time results.

Johnson and Lieberman (1996) investigated closed-form models of golf-ball impacts against a rigid steel surface. Parameters for the models were found from static and dynamic tests. While their results agreed with experimental data, their model did not include a cylindrical impact surface needed for bat studies.

The foregoing represents substantial numerical and experimental contributions in our progress towards simulating solid ball impacts. In the following, a test method will be described to characterize the ball at impact speeds representative of play conditions. The ability of viscoelastic material models to describe the measured impact response of the ball will be examined. To this end, a parametric study of the parameters in the viscoelastic model will be presented to provide a rational guide to ball-model development.

Experiment

Experimental data describing ball response are not widely available. In this paper, a method is presented that approximates the displacement rate and impact force of a softball in play. The experimental results are compared with two numerical models. The first uses DMA to characterize the polyurethane foam core of the softball. The second uses a Power Law viscoelastic model, whose parameters are found empirically from the experimental data.

Instrumented impact

Previous work that considered the load-displacement response of baseballs under quasi-static loading provided an unreasonably low modulus (Shenoy, 2000). The following describes a technique intended to represent the rate and magnitude of deformation that would occur in a ball during play.

Balls were experimentally characterized by measuring forces from rigid wall impacts. Details of the method are presented elsewhere (Smith, 2008). The apparatus consisted of three piezoelectric load cells (PCB model 208C05) mounted between a rigid wall and a solid cylindrical impact surface, as shown in Figure 1. The design is under consideration as an ASTM (American Society for Testing and Materials) test method. It is robust, where the mean stiffnesses of 12 balls measured from two separate fixtures were within 3%. The 57-mm (2.2-inch) diameter cylindrical impact surface corresponded to the bat diameter. Test speeds ranged from 27 to 49 $\mathrm{m \cdot s^{-1}}$ (60 to 110 mph). A representative force–time curve is presented in Figure 2. Integration of the force–time curve provides an impulse that can be compared with ball speeds. Integrating a second time produces a load-displacement curve, as shown in Figure 3.

Dynamic mechanical analysis

Dynamic mechanical analysis is an established method to characterize the rate and temperature dependence of polymers. It involves an applied load (or displacement) in the form of tension, flexure or torsion. The corresponding displacement (or load) is recorded as a function of time and temperature. These can be combined using the time–temperature superposition principle (Flugge, 1975) to infer

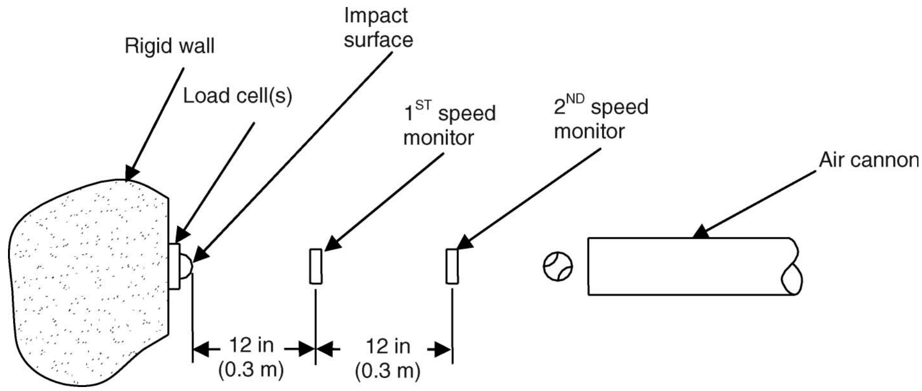

Figure 1. Schematic of test set-up to measure the dynamic response of softballs.

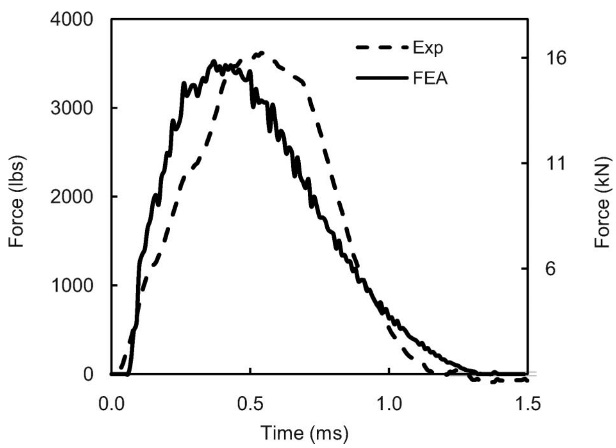

Figure 2. Representative force–time curve from a softball impacting a rigid cylinder at 36 $m \cdot s^{-1}$ (80 mph).

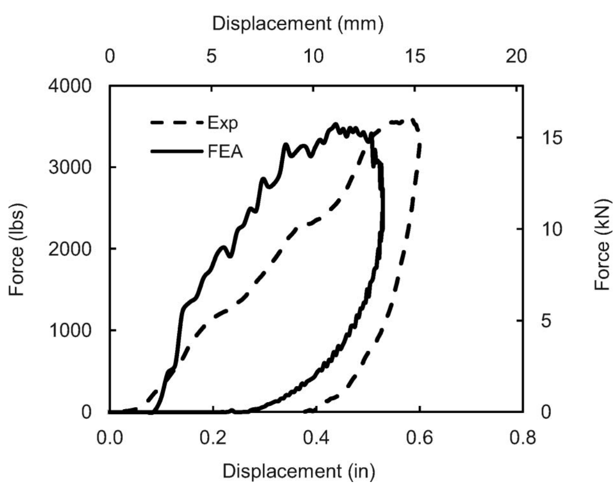

Figure 3. Representative force-displacement curve from a softball impacting a rigid cylinder at 36 $m \cdot s^{-1}$ (80 mph).

deformation rates that are difficult to obtain experimentally.

The softball is made from a polyurethane core that can be sectioned for characterization. The measured rate dependence can be applied to several viscoelastic theories, which can be used in finite element analysis (FEA). Given the generally positive correlation of this technique with golf balls (Mase & Kersten, 2004), its application to softballs was investigated.

Dynamic mechanical analysis was used to obtain the relaxation curve of the polyurethane foam core of the softball. Test coupons measuring $20 \times 6.4 \times 2.7$ mm were machined from the core of a softball with a nominal *COR* and static compression values of 0.44 and 1.67 kN (375 lbs), respectively (ASTM F1887, ASTM F1888). Stress relaxation tests were performed in a three-point bend fixture, where a constant strain was applied to the coupon, while stress was recorded (Menard, 1999). Stress relaxation tests were performed at temperatures ranging from $-60°C$ to $10°C$.

Relaxation curves were combined into a master curve using the time–temperature superposition principle as shown in Figure 4. The curves were shifted horizontally in time to a reference temperature of 22°C using

$$\log a_t = \frac{-c_1(T - T_{ref})}{c_2 + T - T_{ref}}, \quad (4)$$

where a_t is the shift factor, c_1 and c_2 are material constants, T is the temperature, and T_{ref} is the reference temperature (Flugge, 1975). The constants, c_1 and c_2, were adjusted to form a smooth curve and were found to be -2.6 and $-112°C$, respectively. The master curve was fitted to a six-term Prony series using a multi-parameter least squares fit, as shown in Figure 4.

The bend fixture used in the DMA work measured the Young's modulus, while the input for the numerical model was a shear modulus. The two moduli are related according to

$$G = \frac{E}{2(1+v)}, \quad (5)$$

where v is the Poisson ratio. While the Poisson ratio of the polyurethane softball is not known, it is relatively small. The fit presented in Figure 4 illustrates the relatively small contribution of different Poisson ratios.

Finite element model

The fixed cylinder impact described above was modelled using the dynamic finite-element code LS-DYNA (Version 970, LSTC, Livermore, CA). The softball was modelled as an isotropic, homogeneous sphere. Most softballs are made from a 94-mm diameter polyurethane core with a 0.5-mm thick leather or synthetic cover. While the numerical model provided a good approximation of a softball, the homogeneous assumption has also been applied to baseballs that have a non-homogeneous construction. Homogeneity is used out of necessity, because the dynamic responses of yarn and leather are not known. The distinction is not important, however, as the objective of the ball model is to describe its dissipated energy and impact force, for which a homogeneous sphere is a good approximation.

The softball was modelled using eight-noded solid elements and two symmetry planes. Given the dependence of the contact area inherent with spherical impacts, a fine mesh is desirable. Mesh refinement is limited, however, by the computational overhead needed for implicit dynamic simulations. The computational cost of refinement is magnified when a bat is included, where only one plane of symmetry exists.

Ball *COR* was compared with the number of elements ranging from 112 to 7168 for the quarter ball. The model appeared to converge after 2000 elements, although a 1% change still occurred between 4802 and 7168 elements. A model employing 2816 elements for the ball, shown in Figure 5, was used for the following comparisons. While the effect of mesh refinement appears small, the ball properties reported herein (and elsewhere) should not be taken independent of the mesh. A small sensitivity was observed, for instance, with the alignment of the nodes of impacting surfaces with no change in mesh refinement. Nevertheless, the ball model represents play conditions, given the similarity of its characterized response with the deformation that occurs in play.

Prony series model

An impact of 27 $m \cdot s^{-1}$ (60 mph) was simulated using the six-term Prony series model. The one-dimensional coefficients of equation (3) were taken from the DMA of the polyurethane core (Duris, 2005). Because the transverse response was not measured, representative values of the bulk modulus (0.20 Msi or 1.4 GPa) and Poisson's ratio (0, 0.25, 0.50) were used. The *COR* was double the nominal value for all three Poisson ratios. The peak impact force was nearly double that measured experimentally. While the impact force decreased with modulus (increasing Poisson's ratio), it was still 60% higher than the experiment at the limit, $v = 0.50$.

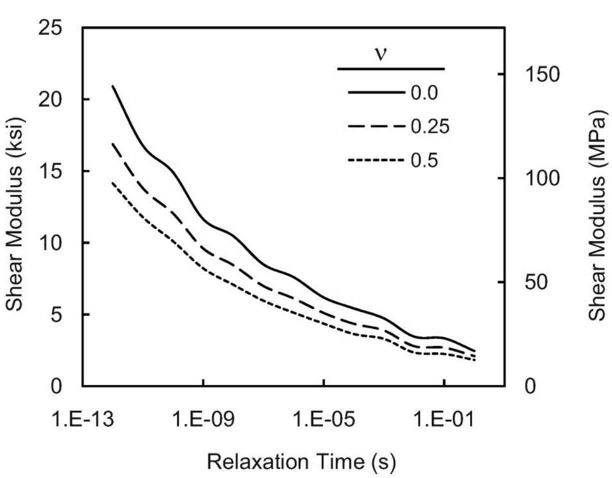

Figure 4. Prony-series master curve for three values of the Poisson ratio.

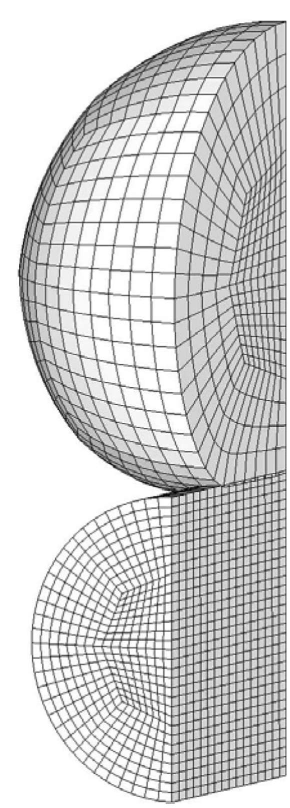

Figure 5. Finite element numerical model used in the parametric study.

To improve the correlation, the reference temperature was reduced by 10°C (Mase & Kersten, 2004), which reduced the *COR* slightly (2%) and increased the impact force substantially (14%). The Prony series was also examined as a function of speed, from 27 to 49 $\mathrm{m \cdot s^{-1}}$ (60 to 110 mph). The *COR* and impact force increased slightly (2.5%) and substantially (100%), respectively. While the impact force increase agreed with the experiment, the *COR* change did not.

The poor correlation of the DMA characterized softball model was disappointing, but not surprising. DMA testing is done in the linear range of the polymer. Bat–ball impacts involve large strains that are three orders of magnitude higher than that used in DMA characterization. In the case of polyurethane core softballs, non-linear softening occurs at large strains characteristic of impact. Apparently, the small strains used in DMA characterization are not sufficient to describe the softball under play conditions.

An advantage of DMA is that it provides systematic techniques to describe the polymeric response over a wide range of displacement rates. In light of its apparent shortcoming for the current application, a method was sought that could more closely replicate the ball-deformation magnitude in play conditions. The instrumented rigid-cylinder impacts were modelled numerically, and the viscoelastic parameters were iteratively adjusted until the observed response was replicated. Because the range in displacement rates of softball impacts is narrow (compared with Figure 4), a simpler three-term Power Law viscoelastic model was selected.

Power Law model

In the Power Law model (equation 1), stiffness is governed by G_0 at short and by G_∞ at long durations. The time constant, β, determines how quickly the modulus changes between the two values. The bulk modulus, k, is constant, producing a time-dependent Poisson's ratio, v, according to

$$v(t) = \frac{3k - 2G(t)}{6k + 2G(t).} \qquad (6)$$

Equation (6) places bounds on some of the model parameters as the Poisson ratio must lie between 0 and 0.5 for all time. The Power Law parameters were varied in a systematic fashion (as described below) to identify combinations of parameters that could describe the measured ball *COR* and impact force. Even with the few parameters used in the Power Law, the solution was not unique. In one exercise, ten combinations of Power Law parameters were identified that described the measured ball response (Duris, 2005).

A representative impact force of the model is compared with the experiment as a function of time and displacement in Figures 2 and 3, respectively. While impulse and contact duration were in good agreement (within 0.5% and 9%, respectively), the numerical models consistently demonstrated greater stiffnesses than those measured experimentally. The high stiffness of the FEA models over the initial 1.5 mm of displacement is likely due to the surface contact algorithm. When penetration occurs, surface forces are applied to the contacting regions. The forces are applied iteratively until the penetration is minimized. This causes an initial low stiffness when contact first occurs and the resistive forces are not developed, followed by high stiffness as the penetration is minimized.

The higher stiffness of the FEA model beyond 1.5 mm of displacement could be related to the different mechanisms dissipating energy in each case. The numerical model, for instance, does not generate heat upon impact as has been observed experimentally (Duris & Smith, 2004). The polyurethane core of the softball likely undergoes non-linear softening at high deformation magnitudes. The viscoelastic properties for the linear Power Law model used here represent mean ball responses over its linear and non-linear ranges. Because the bat is not rate dependent, the difference in ball stiffness between the experiment and the numerical model should not affect bat–ball performance simulations, or the outcome of this work.

Parametric study

To gain insight into parameters in the Power Law viscoelastic model, a parametric study was conducted in which we considered their influence on the ball *COR* and impact force. To simplify the comparison, impact forces were normalized by the forces obtained for the nominal case, F_n, described below. The mass density was kept constant at 415 $\mathrm{kg \cdot m^{-3}}$. In the parametric study, the following nominal parameter values were used: $k = 5.5$ GPa (0.80 Msi), $G_0 = 0.19$ GPa (28 ksi), $G_\infty = 10$ MPa (1.5 ksi), $\beta = 6.8 \times 10^4$, $v_{in} = 42$ $\mathrm{m \cdot s^{-1}}$ (95 mph).

The instantaneous modulus, G_0, is considered in Figure 6. At low G_0, the difference between G_0 and G_∞ was small, which resulted in a nearly elastic response and a nominal impact force. The dissipated energy increased (i.e. decreasing *COR*) with G_0 until a minimum was reached. An elastic response was again approached as G_0 was increased further, where the instantaneous modulus governed the response during contact more than time. Softball and baseball numerical models usually have a G_0 that is on the left side of the curve that tends to affect *COR* more than the impact force.

The long-term modulus is considered in Figure 7. Both the *COR* and impact force were observed to increase with G_∞. As G_∞ approached G_0, the response became elastic. The impact force was observed to increase with stiffness, while the contact duration decreased.

The time constant, β, is considered in Figure 8. A minimum, similar to Figure 6, was observed for *COR*, while the impact force decreased in a step fashion with increasing β. At low values of β, the response did not change appreciably during contact and was governed by G_0, which minimized the dissipated energy and produced a high impact force. At high values of β, the response changed to G_∞ before appreciable deformation occurred. Energy loss was again minimized, and the impact force decreased as G_∞ decreased. The magnitude of the left and right force plateaus was governed by G_0 and G_∞, respectively.

The bulk modulus, k, is considered in Figure 9. The Poisson ratio will increase with the bulk modulus according to equation (6). Because the Poisson effect is a secondary response, the bulk modulus was anticipated not to have a strong effect on ball responses. The *COR* decreased with increasing k, although at a low rate. The deformation and dissipated energy should increase with the Poisson ratio. The impact force was nearly unchanged with increasing bulk modulus. Increased transverse deformation with Poisson's ratio should not appreciably affect the material stiffness or impact force.

The incoming speed, v_{in}, is considered in Figure 10. The *COR* decreased with speed. Ball deformation (and dissipated energy) should increase

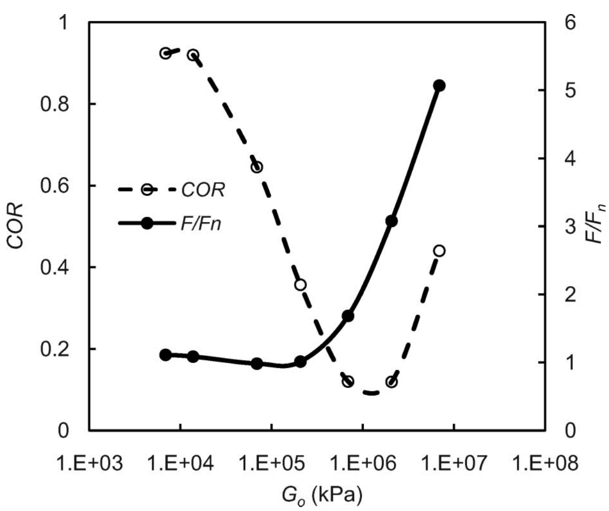

Figure 6. Effect of G_0 on ball *COR* and the normalized impact force.

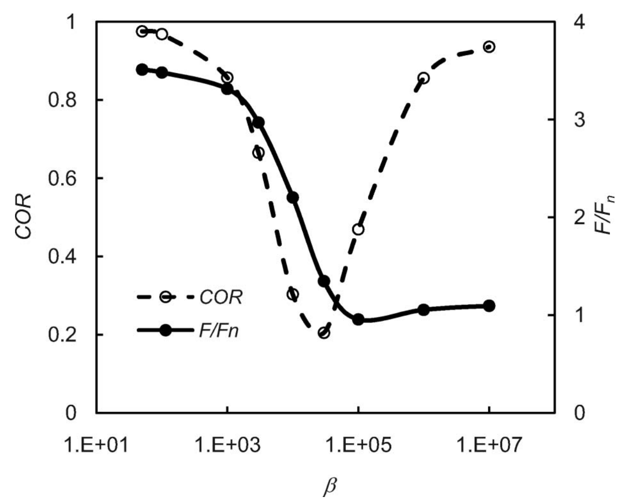

Figure 8. Effect of β on ball *COR* and the normalized impact force.

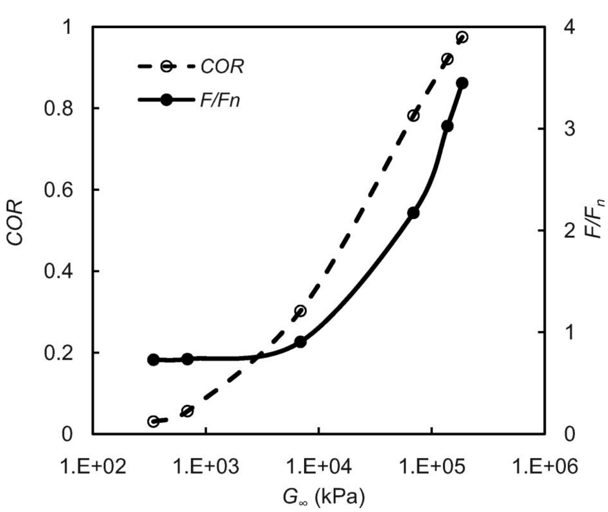

Figure 7. Effect of G_∞ on ball *COR* and the normalized impact force.

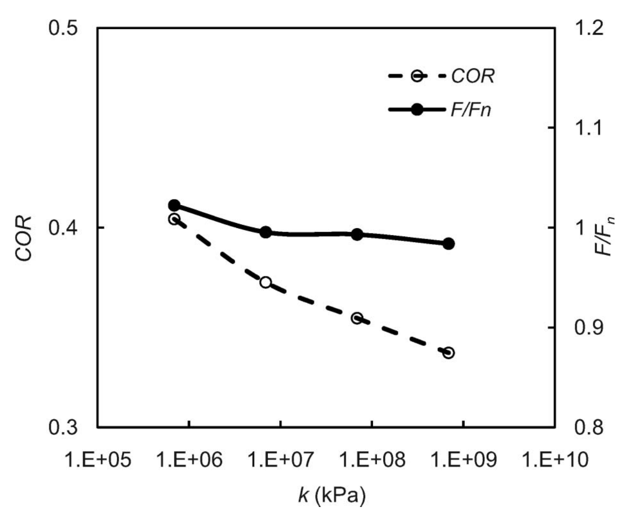

Figure 9. Effect of k on ball *COR* and the normalized impact force.

with the impact speed. The impact force increased with speed, which was due to the increased impulse needed to change the ball direction with incident ball speed.

Effect of impact speed and surface geometry

The impact force and *COR* from the model are compared with the experiment as a function of speed in Figure 10. Each data point is the average of six balls. The error bars represent the standard deviation of each set. Over the range of speeds considered here, predictions of *COR* are in excellent agreement with the experiment (root mean square error of 0.004 or 1%), while the predicted effect of speed on force was slightly higher than in the experiment (root mean square error of 2.1 kN, 474 lbs or 9%). The comparisons are encouraging given the simple linear viscoelastic model used in this work. While a variety of combinations of viscoelastic properties were found that described the ball response at a given speed, they all exhibited a similar dependence on speed.

Balls are often tested by impacting a rigid flat plate. For a given impact speed, a flat wall will produce a larger contact area than the cylindrical surface considered above. The increased contact area will lower the ball deformation and dissipated energy (higher *COR*). A larger contact area will also produce a higher effective ball stiffness and impact force. Six softballs were impacted at 40 $m \cdot s^{-1}$ (90 mph) against a rigid flat plate and the cylindrical surface. The ratio of the flat to cylindrical impact force and *COR* is presented in Figure 11. The flat surface produced mean forces and *COR* that were 15% and 3% higher, respectively, than those from the cylindrical surface.

The flat and cylindrical surfaces were compared in the numerical model to determine how the ball model adjusted to different impact surfaces. The results are included in Figure 11. The surface effect on force was in good agreement with the experiment (17%), while the predicted *COR* showed a substantial decrease (13%) compared with the measured small increase.

Although simple viscoelastic models could independently control energy loss and hardness, rate effects were difficult to control and changes in impact geometry sometimes yielded incorrect trends. Until improved models are developed, it is recommended that ball models should be characterized from experimental data at deformation rates, impact forces, and surface geometries that closely resemble the conditions of interest.

Summary

Two methods that characterize dynamic responses of a softball have been considered. While dynamic mechanical analysis could describe viscoelastic responses of the softball core over a wide range of displacement rates, the numerical model incorporating these responses did not dissipate sufficient energy during impact. An improved comparison between the numerical model and experiment was obtained by empirically fitting viscoelastic parameters with an instrumented impact surface. A parametric study describing the sensitivity of the viscoelastic parameters was presented, which showed that dissipated energy and ball hardness could be controlled independently. It was observed, however, that the numerical ball model produced slightly greater stiffness than the experiment. The correlation with the experiment also tended to deteriorate when the

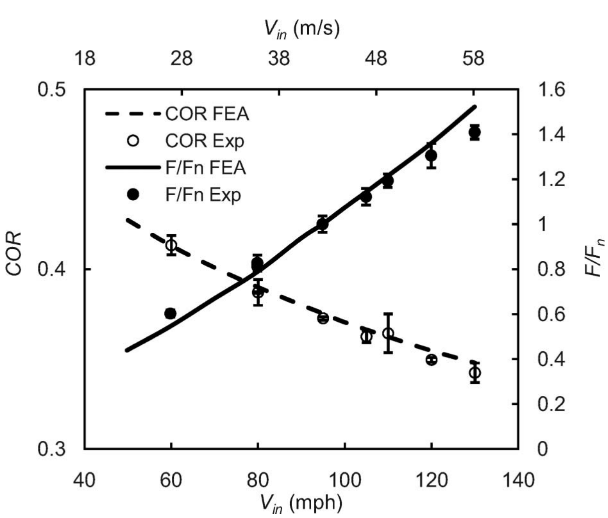

Figure 10. Effect of v_{in} on ball *COR* and the normalized impact force.

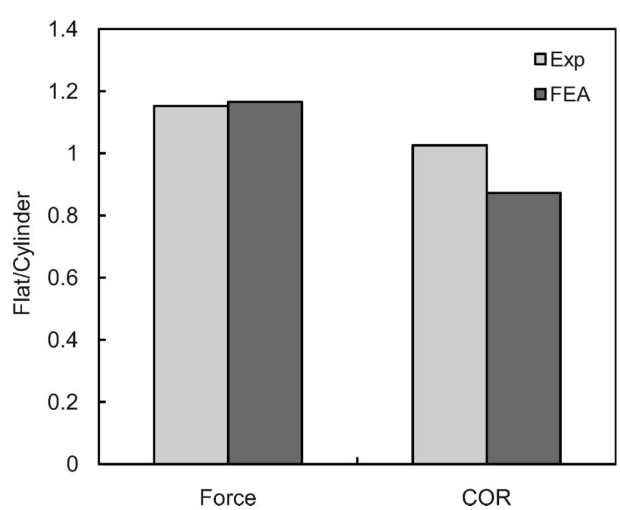

Figure 11. Effect of geometry on the impact force and *COR*.

surface geometry was changed from cylindrical to flat. It is recommended, therefore, that numerical ball models be developed from experimental data that closely replicate the intended use of the model.

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